THE JOURNAL OF ALTERNATIVE AND COMPLEMENTARY MEDICINE
Volume 3, Supplement 1, 1997, pp. S-89–S-99
Mary Ann Liebert, Inc.

Mathematical Modeling as a Tool for Basic Research in Acupuncture

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ABSTRACT

We propose and illustrate a five-step strategy for basic research of traditional explanatory frameworks of acupuncture. Our approach is based on using mathematical models as a bridge between the traditional explanatory frameworks of acupuncture and Western research technologies. *Step 1.* Carefully select and document case studies that would allow simultaneous interpretation within several "traditional" explanatory frameworks of acupuncture. *Step 2.* Develop minimal theoretical models connecting the diagnosis and treatment within the context of each of the selected explanatory frameworks of acupuncture. *Step 3.* Develop minimal dynamical systems models for each theoretical model from Step 2, so as to tighten their logical structure and to bring them into a falsifiable and more abstracted format (which provides the link between the theoretical models at Step 2 and possible Western-based models, and hence Western measurement technology). *Step 4.* Interpret the mathematical models from Step 3 within the framework of "Western" scientific perspectives. *Step 5.* Select and conduct appropriate "objective" (skin electrical impedance, etc.) measurements to test the validity of the models at Step 4. Repeat Steps 1–5 as needed.

INTRODUCTION

Acupuncture, the use of needles, and related ancillary techniques to regulate physiology and relieve symptoms has been practiced in China for more than 2000 years, and in Japan and Korea for more than 1450 years. It arrived in Europe approximately 300 years ago and in North America more than 150 years ago (Birch and Felt, 1997; Needham, 1980), recently spreading to most countries around the world (Cheng, 1987; WHO, 1985). The earliest major documents on acupuncture are revered as the primary sources of acupuncture and have been commented on by numerous authors over the centuries (Sivin, 1987; Ishihara, 1983; Unschuld, 1986). For many acupuncturists today, these texts are still highly regarded, studied, and provide the basis for much of modern practice. Frequently the authenticity of a particular tradition or practitioner's approach is claimed by citing appropriate passages from these ancient and later texts (Connelly, 1979; Kaptchuk, 1983; Maciocia, 1989; Matsumoto and Birch, 1988;

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Shudo, 1990; Soulie de Morant, 1994). Differences within (Epler, 1980) and between (Birch, 1992) these texts and sociopolitical and cultural variations in each country where acupuncture has been practiced have fostered a wide variety of conceptual models, methods, and techniques of practice (Birch and Felt 1997; Needham, 1980). The use of a single term "acupuncture" can be misleading as it implies a coherent and uniform model of practice, which is not the case. Not only is there historical and modern evidence of the use of a wide range of treatment techniques (Birch, 1997a) and diagnostic assessment methods (Birch and Felt, 1997; Birch, 1992, 1997a), but the conceptual frameworks by which acupuncture has been practiced also show considerable variety, both in traditionally based and modern frameworks. There were many historical systems (Birch and Felt, 1997; Matsumoto and Birch, 1988, 1986; Sivin, 1987; Unschuld, 1986) and today there are many traditionally based systems of acupuncture (TBSAs), such as traditional Chinese medical (TCM) acupuncture (Cheng, 1987; Kaptchuk, 1983; Maciocia, 1989), traditional or five-element acupuncture (Connelly, 1979), pre-TCM acupuncture (So, 1987; Soulie de Morant, 1994), meridian therapy (Shudo, 1990), Yin-yang channel balancing therapy (Manaka et al., 1995), medical acupuncture (Helms, 1995), and many modern systems of practice (Baldry, 1989; Nakatani and Yamashita, 1977; Ulett, 1992).

Each TBSA has its unique methods of assessing patients, deciding what the problem is, and deciding which acupuncture points and techniques to use. As we will see, were the same patient to be assessed by practitioners of different TBSAs, the labels and treatments would all be different (Birch, 1995; Birch and Felt, 1997). What the systems all have in common is the idea that the treatment of specific points using specific techniques for each individual patient can maximize the efficacy of treatment. This is interesting because in each TBSA, as long as a diagnosis is arrived at, a treatment automatically follows (Birch, 1997b; Birch and Felt, 1997). It thus appears that the traditional concepts on which each system is based, and their associated assessment methods ultimately guide the determination of what points to treat with what techniques. This leaves the researcher who is interested in testing acupuncture with many questions about the nature of these TBSAs. Because each traditional framework and its associated methods of practice guide the practitioner in their selection of treatment points and associated techniques for each patient, then either: (1) the explanatory frameworks are nothing but heuristic devices or techniques that give the practitioner guidelines for selecting treatment points and techniques, in which case all the traditional concepts—Qi, jing luo, zang fu, yin yang, etc., may have no basis in physical measurable reality or (2) the explanatory frameworks do refer to some phenomena in nature that the various assessment and treatment methods take advantage of to treat patients, in which case the traditional concepts may have an as yet untested, unmeasured basis in physical reality.

According to practitioners of TBSA approaches, the individualization of treatment based on traditional concepts and practices produces greater treatment effects than when acupuncture is used without these methods. However, because no clinical trials of acupuncture have been conducted that compared the various forms of acupuncture with each other, or have even tested TBSA approaches (Birch, 1997b), there is no basis for judging if this claim is correct. Furthermore, however, no studies have been adequately designed that could systematically explore and test these traditional frameworks, their concepts and clinical methods, it is appropriate to begin designing studies that allow us to test such claims. Dismissing these claims as nonsense (Ulett, 1995) may reflect cultural bias and an inadequate grasp of the subject matter more than it does good scientific judgement. However, insisting that the paradigm of Western (reductionist) research cannot be used to test the "holistic" paradigm of acupuncture (Patel, 1987), is not only a gross simplification of the nature of acupuncture (Unschuld, 1987, 1992), but the argument makes faulty assumptions about the nature of the so-called paradigms (Birch, 1997a; Vickers, 1996). Appropriate studies have not yet been designed that allow us to make a scientific judgment one way or the other (Birch, 1997b). Essentially the basis of TBSAs can be summarized as in Figure 1.

In order to test this hypothesized intermediate modulating system, the authors propose a step-by-step approach:

Step 1: carefully select and document case studies that would allow simultaneous interpretations within several TBSA approaches.

Step 2: develop minimal theoretical models connecting the diagnosis and treatment within the context of each of the selected TBSA frameworks.

Step 3: develop minimal dynamical systems models for each theoretical model from Step 2, so as to tighten their logical structure and to bring them into a falsifiable and more abstract format (which provides the link between the theoretical models at Step 2 and possible Western-based models, and hence Western measurement technology).

Step 4: interpret the mathematical models from Step 3 within the framework of "Western" scientific perspectives.

Step 5: select and conduct appropriate "objective" (skin electrical impedance, etc.) measurements to test the validity of the models at Step 4.

Repeat Steps 1 to 5 as needed.

The remainder of this article is organized as follows. In the first section we present a model case study and the treatments according to three TBSAs. In the next section, we develop a simple (mathematical) model of disease that provides us with a coherent framework for implementing Step 2. We then address Step 3 (see Birch and Friedman, 1994, 1989; Friedman et al., 1989a, 1989b, for our earlier work related to steps 2 and 3.) In the next section, (Steps 4 and 5), we outline an approach that links the TBSA mathematical models from Step 3 with electrodermal measurements. The final section contains conclusions.

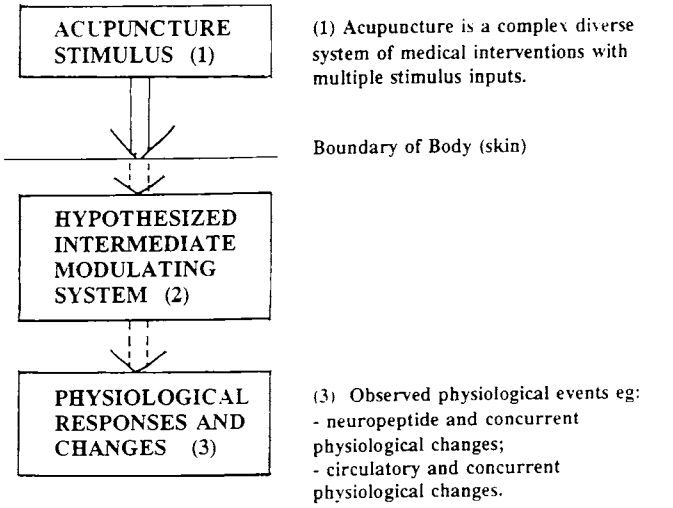

FIG. 1. The theoretical basis of TBSA.

A MODEL CASE STUDY: THREE TREATMENTS

Examination

Female age 37. Main complaints: cystitis, dysmenorrhea, and chronic low back pain. *Visual inspection:* yellowish around the eyes; thin tight and muscled frame; thin, red tongue body with yellow fur at the rear of the tongue; presence of pigmentation in the interscapular region with moles on the sides of the neck and abdomen; vascular spiders visible on the costal region over the liver. *Olfactory and auditory inspection:* a slightly-sickly sweet body odor; a singing-like, lilting tone of voice. *Questioning:* dislikes hot weather; the low back pain is better with pressure applied to the back; hands generally feel very warm; feels tired and hot in the late afternoon; is generally very pensive and easily irritated; the urine flow is urgent, painful, obstructed, and quite yellow in color; the menstrual flow is heavy with some pain and clotting; sometimes has a problem with sweating at night. *Palpation:* the second right deep pulse is the weakest, the second and third left deep pulses are also quite weak; the pulse is overall deep, thin and fast; the abdomen exhibits stiffness in the right subcostal region and above the navel, with pressure pain and stiffness in the areas immediately to either side of the navel and in the area about 2 inches to the left of the navel, extending below the level of the navel.

As Step 1, the Table 1 summarizes the (1) TCM acupuncture (Cheng, 1987; Kaptchuk, 1983; Maciocia, 1989); (2) meridian therapy (Shudo, 1990); and (3) Yin-Yang channel balancing treatments (Manaka et al. 1995; Mat-

sumoto and Birch, 1988) that would be given to the same patient, described above, were she assessed and treated by practitioners of each TBSA. This case is described in detail elsewhere, (Birch 1995; Birch and Felt 1997) but sufficiently illustrates three TBSA approaches that will be further explored in Steps 2–4.

THEORETICAL RATIONALE FOR THE PRESCRIBED TREATMENTS. A SIMPLIFIED MATHEMATICAL MODEL OF DISEASE

Each treatment framework has a somewhat different rationale for how to assess and treat the patient. In these examples, each is based at least in part on some of the traditional concepts and methods that date back some 2000 years. In order to gain a clear overview of each treatment framework, it is necessary to grasp the basic components of the TBSAs themselves.

Essentially, the historical explanations of human physiology, pathology, diagnosis, and treatment are based on the concept of Qi, which is variously translated as influences, finest matter influences, and more popularly and less accurately as energy or vital energy. In relation to life and physiology, this Qi is said to take many forms, have many functions, circulate in the body, be derived from a reservoir given at birth and from assimilation from the environment after birth through breathing and digestion. In some representations of this concept, the Qi is a seamless whole, where all of its aspects at least within the body are affected by and affect all others. The Qi circulates along pathways, primarily the jing mai or channels, with a variety of secondary channel systems. It is produced in the processes of assimilation, and transformation of stored aspects by the zang-fu or organs. The jing mai and zang-fu are interconnected in an intricate system of processes and relationships. General theoretical constructs such as Yin-Yang and the wu xing or five phases are described with reference to regulatory principles that govern all aspects of the Qi system, and also label-categorize all of those aspects and their derivative structures as the organizing principle or skeleton onto which the body of Qi systems is hung. Qi is expended throughout life by normal physiology and activities and in response to disease processes. Disease processes are often labeled with reference to which aspects of the overall structure are awry: whether it is not circulating correctly, whether it is not being produced correctly, whether too much of it is being expended fighting pathogens and disease processes.

Here is the simplest description of the condition and content of Qi in the body:

$$Q = Q_p + Q_e + Q_{cd}$$

Here Q = total amount Qi in the person or an organ/functional unit (+ related substances such as blood), Q_p = production of Qi (respiration, digestion, assimilation, conversion of reserves, transformation of stored substances) Q_e = expenditure of Qi (activities, excretion, and normal physiology + fighting disease [pathogenic Qi]) Q_{cd} = circulation and distri-

TABLE 1. SUMMARY OF (1) TCM ACUPUNCTURE, (2) MERIDIAN THERAPY, AND (3) YIN-YANG CHANNEL BALANCING TREATMENTS IN A MODEL CASE STUDY

TBSA	*Points*	*Technique*	*Depth*	*Subjective Sensation*
(1)	KI-3, BL-52	Supplement	7.5–37.5 mm	*de qi*
	CV-4, SP-9, BL-28, BL-40	Drain	12.5–37.5 mm	*de qi*
(2)	LV-8, KI-10	Supplement	1–2 mm	none
	CV-4, BL-18	Supplement	1–4 mm	none
	BL-23	Supplement	1–4 mm	none
	SP-3	Drain	1–2 mm	none
(3)	PC-6, SP-4 TB-5,	IP cord	2 mm	none
	GB-41	IP cord	2 mm	none
	BL-18, BL-23	needle-moxa	10–15 mm	heat
	BL-28	needle-moxa	15 mm	heat

See text for references.

bution of Qi (the whole channel system, both primary and secondary systems).

Different TBSAs draw on different components of this simple equation, and, assuming the seamless interconnectedness of the whole system of Qi, each assumes an impact on each of the unstated components. For example, it is possible to view (1) the *ba gang bian zheng* or traditional Chinese medical acupuncture diagnosis and treatment simply in terms of weakness of Qi production Q_p, (kidney yin vacuity) and Qi expenditure, Q_e to process, eliminate the pathological factors (damp heat in lower jiao); the circulation and distribution of Qi Q_{cd} is assumed to be adjusted once these other factors are corrected. In the same vein, (2) the *keiraku chiryo* or Japanese channel therapy acupuncture diagnosis and treatment can be viewed in terms of a problem of the circulation and distribution of Qi, Q_{cd} (liver vacuity and spleen repletion); the production (Q_p) and expenditure (Q_e) of Qi are assumed to be adjusted once these circulation problems are corrected. (3) The "Yin-Yang channel balancing therapy" acupuncture diagnosis and treatment can also be viewed in terms of the circulation and distribution of Qi, Q_{cd} (cross syndrome, liver and blood stasis); the production (Q_p) and expenditure (Q_e) of Qi are also assumed to be adjusted once the circulation problem is corrected.

This simplified view of the complex events and phenomena that have been historically described in relation to health, physiology, and disease can be used to picture the diverse frameworks used in modern practice.

A further issue of importance in grasping these diverse frameworks of diagnosis and treatment is that of explaining how the treatments work (that is, in terms of these historical concepts). Again, we have a simplified model that we believe can capture the salient points of each framework.

In the interpretation of a typical practitioner, the traditional frameworks have as a basic assumption, the concept of health as a "balance" of all the diverse components that comprise the traditionally formulated frameworks. Because all of these components have Yin-Yang and five-phase correlates, we find traditional formulations of health in terms of a kind of "Yin-Yang balance" and"five-phase balance." Disease, on the other hand, is seen as a process where "unbalanced" conditions have developed. In a "healthy" balanced condition, disease-causing factors can be resisted and overcome. In a diseased unbalanced condition, these factors are more likely to create more disease, making the condition more "unbalanced."

We believe that the terms "balance" and imbalance are modern simplifications of the TBSAs. We use instead what we believe to be a more precise interpretation of health and disease in terms of stable states. When an individual is healthy they are in a stable state, by which we mean that the relevant physiological parameters are in the normal range and return to this normal range if slightly perturbed. When an individual is diseased, they are in another stable state where relevant physiological parameters are in the abnormal range and return to this abnormal range if slightly perturbed. We recognize that the abnormal physiological parameters of the diseased state are assessed by practitioners of the various TBSAs. In some, such as the *ba gang bian zheng*, the pathological labels primarily focus on an alternate model of physiology and its imbalance; in others, such as the *keiraku chiryo* system, the pathological labels primarily refer to conditions that are out of balance causing a breakdown of the normal regulation of physiology. Practitioners of these traditional frameworks of practice use the looser terms of "Yin-Yang balance" or "five-phase balance" to refer to their assessments of the patient, without clearly stating what physiological parameters are included in this. As was suggested earlier, these traditional frameworks may only be heuristic devices that organize treatment point selection. This is the question to be investigated.

Traditionally and in modern practice, health and the treatment of disease are formulated both in terms of these global body perspectives (Qi, Yin Yang, five phases) and the more localized disease symptoms themselves. Two basic approaches to treatment are typically used, the *zhibenfa*, targeting the overall health of the patient, expressed in terms of Qi, Yin Yang, the five phases, etc., and the *zhibiaofa*, targeting the relief of specific symptoms. In essence the health of a patient refers to a combination of two factors:

(1) The overall health state of the patient as judged by the four diagnostic inspections as used in the case above. We lump them together and denote this combined health-state variable as h. We further make a convention that when these various categories lumped together are overall close to normal, we say that h is high, and h is low when they are far from normal.

(2) We next introduce a parameter λ (lambda), which accounts for the patients' ability to recover from disease and resist it. If λ is high, this ability is poor, if λ is low, this ability is good. We represent this diagrammatically in Figure 2.

Here the "health region" is between λ and 1; it is small in case 1, and it is large in case 2. The disease region is between 0 and λ; it is large in case 1, and it is large in case 2. In each region the arrow points to the direction of change of health in time: when the initial symptoms h_0 are in the disease region, they will become worse with time (the arrow points to the left); when the initial symptoms h_0 are in the health region, they will improve with time (the arrow points to the right). As another way of understanding this, imagine that the line between 0 (poor health) and 1 (good health) is a cross section through a hill where λ represents the highest point on the hill, and 1 and 0 are at the same level. If a ball is placed to the left of λ it will naturally roll down towards 0, likewise if the ball were placed to the right of λ it would roll down towards 1.

With the same initial symptoms, h_0, in both cases, in case 1 (large disease region) the disease will progress, whereas in case 2 (small disease region) the patients' health will improve with time. The patient in case 1 will generally be unhealthy, typically with many minor health complaints, none of which progress rapidly to a serious stage, but generally their condition will progress. The patient in case 2 will generally be very healthy, and when symptoms do develop they are quickly resolved.

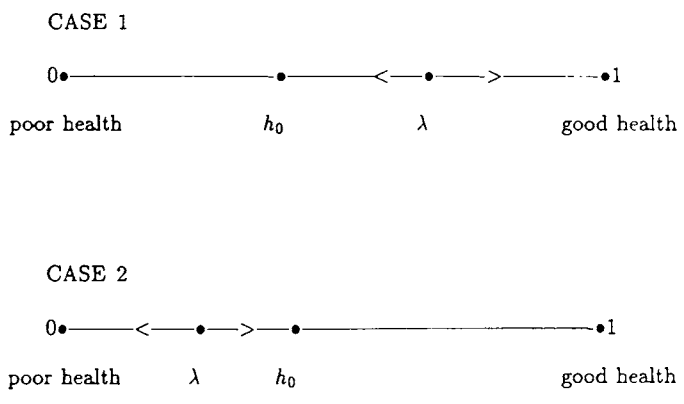

FIG. 2. A graphic representation of the health dynamics described by eqs (1) below.

The above intuitive description of health dynamics can be easily formulated in a more precise form using the (mathematical) language of dynamical systems as follows.

We assume that the organism can be subdivided into separate (functional) units. We let $h(t)$ be a (generic) state variable (= lumped together symptoms) that accounts for the health state of a functional unit. Let $Q(t)$ be the principle state variable that refers to the amount of Qi in a particular unit as a whole (below, we will use subscripts, when we want to specify these units or types of Qi). We are interested in describing effects of $Q(t)$ on the evolution of $h(t)$ in time t.

For simplicity, we assume that a functional unit can be in either of the two homeostatic states: a diseased one or a healthy one (this implies that the organism adapts to a diseased condition, which is often observed in practice). Mathematically we describe these as two stable steady states $h(t) = 0$ and $h(t) = 1$ of the function $h(t)$. Then the process of change of health in time is described by the following differential equations:

$$\begin{aligned} &\frac{dh}{dt} = h(\lambda - h)(1 - h),\ h(0) = h_0,\ 0 < t, \\ &\frac{d\lambda}{dt} = f(Q),\ 0 < t. \end{aligned} \tag{1}$$

The first equation describes the dynamics of $h(t)$. It is easy to show that $h = 0$ and $h = 1$ are the two stable steady states. In particular, the value of parameter λ determines whether the functional unit becomes "more healthy" or "more diseased" with time and how fast. Specifically, if $h_0 < \lambda$, then the initial state of the patient is in the disease region, which means that it deteriorates with time: $h(t) \to 0$, as $t \to \infty$; while if $h_0 > \lambda$, then the initial state

of the patient is in the health region, which means that it improves with time: $h(t) \to 1$, as $t \to \infty$. The second equation accounts for the effect of Qi on the health dynamics. If $f(Q) > 0$ then $\lambda(t)$ increases with time, decreasing the health region, while $f(Q) < 0$ would lead to the increase of the health region.

A typical healing process can be described as follows. Suppose that initially $0 < h(0) < \lambda(0)$, i.e., $h(t)$ is in the disease region. Then after the treatment (or a series of treatments) at the time t_1 $\lambda(t) - h(t)$ crosses 0, and from now on $0 < \lambda(t) < h(t) < 1$: $h(t)$ is in the health region, i.e., the patient's health starts to improve, and this process continues.

We saw that in acupuncture, there are two basic treatment approaches: treating to correct the perceived problems and their causes (Qi system), and treating to relieve symptoms. In simple terms, Qi-based treatments target increasing the health region (lowering λ), and symptom-relief treatments target improving health in the sense of relieving the symptoms (increasing the health state variable $h(t)$). Our simple mathematical model accounts for a variety of clinically observed phenomena. For example, consider a patient, whose symptoms improved after a treatment, but after a while worsened again. Explanation: Initially, $h(0) < \lambda(0)$, where $\lambda(0)$ is large enough. After treatment, though $h(t) > h(0)$, but λ did not significantly change, and we still have $h(t) < \lambda$. Hence the patient's state will start deteriorating again.

Note that this simple mathematical model of health and healing capabilities does not depend on particular treatment methods or conceptual and clinical frameworks. It can therefore be used as a tool to link the diverse treatment approaches in the different TBSAs, thus providing a unified perspective to account for some essential elements in these TBSAs.

SOME MINIMAL DYNAMICAL SYSTEMS MODELS FOR THE MODEL CASE STUDY

Ba gang bian zheng: traditional Chinese medical acupuncture

A simplified interpretation of the diagnosis: kidney yin vacuity means that the production rate, call it q_p of Qi is too low; damp heat in the lower jiao is a disease, that requires increase expenditure rate of Qi, call it q_e.

A mathematical model for the treatment is as follows. Let t_0 be the time duration of the treatment, and T be the time, during which the effects of the treatment last. We can assume that t_0 is much less than T ($t_0 << T$). Assume also that all quantities in the equations below are positive.

$$
\begin{aligned}
&\frac{dq_p}{dt} = b_p,\ q_p(0) = q_{p0},\ 0 < t < t_0,\\
&\frac{dQ_p}{dt} = a_p q_p,\ Q_p(0) = Q_{p0},\ 0 < t < T,\\
&\frac{dq_e}{dt} = b_e,\ q_e(0) = 0,\ 0 < t < t_0,\\
&\frac{dQ_e}{dt} = a_e q_e,\ Q_e(0) = Q_{e0},\ 0 < t < T,\\
&Q(t) = Q_p(t) - Q_e(t),\ 0 < t < T.
\end{aligned} \tag{2}
$$

The first equation describes increasing, during the time interval $[0,t_0]$ of the treatment, the production rate of Qi due to supplementing KI-3, BL-52, CV-4. The second equation describes the process of production of Qi. The third equation describes turning on, during the time interval $[0,T_0]$ of the treatment, the process of expenditure of Qi (to counteract the disease) due to draining SP-9, BL-28, BL-40. The fourth equation describes the process of expenditure of Qi. The fifth equation is the balance equation of Qi.

The healing process is described by the differential equation:

$$
\frac{d\lambda}{dt} = -Q_e,\ 0 < t < T. \tag{3}
$$

Since $d\lambda/dt < 0$, Qi is spent for increasing the size of the health region; see equation (1).

Keiraku chiryo: channel therapy

In this case the circulation-distribution Q_{cd} system is problematic. A simplified diagnosis: liver and kidney vacuity, which means that the levels Q_c and Q_m of the liver (child) and kidney (mother) Qi, respectively, are too low.

A mathematical model for the treatment is as follows. As before, we assume that $t_0 << T$, and that all quantities in the equations below

are positive. A treatment taking advantage of the five-phase interactions in the case of the above channel vacuity would be that of supplementing LV-8 and KI-10. This can be expressed as follows:

(4)

$$\frac{dq_{m,c}}{dt} = b_{m,c},\ q_{m,c}(0) = 0,\ 0 < t < t_0,$$

$$\frac{dQ_c}{dt} = -a_c(Q_c - Q_c^0) + a_{m,c}(Q_m - Q_m^0) + a'_{m,c}q_{m,c},\ Q_c(0) = Q_{c0},\ 0 < t < T,$$

$$\frac{dq_{i,co,m}}{dt} = -b_{i,co,m},\ q_{i,co,m}(0) = 0,\ 0 < t < t_0,$$

$$\frac{dQ_m}{dt} = -a_m(Q_m - Q_m^0) - a_{c,m}(Q_c - Q_c^0) - a'_{co,m}q_{i,co,m},\ Q_m(0) = Q_{m0},\ 0 < t < T.$$

The first equation describes turning on the process of flow of Qi from elsewhere into the child channel by supplementing (applying a positive stimulus $b_{m,c}$ to) the mother point LV-8 located on the vacuous child channel, Q_c, during the time interval $[0,t_0]$ of the treatment. The second equation is the balance equation of Qi in the child channel: on the left we have the rate of change of Qi; the first term on the right accounts for the homeostatic effect of the channel on itself, where Q_c^0 is the normal level of Qi; the second term accounts for the effect of the mother channel on the child channel (according to the creative cycle law); the third term accounts for the treatment effect of supplementing the mother point LV-8 on the child channel (again, according to the creative cycle law). The third equation describes turning on the process of flow of Qi from elsewhere into the mother channel by draining (applying a negative stimulus $b_{i,co,m}$ to) the isophasal point SP-3 on the spleen channel, $Q_{co,m}$. SP-3 is the earth point, and the spleen channel is an earth channel; hence SP-3 is the isophasal (denoted by the subscript i) point of the spleen channel. At the same time, the spleen channel controls the kidney (mother) channel (according to the controlling cycle law). Here we identify the spleen channel by the subscripts "co,m": "controlling mother." The fourth equation is the balance equation of Qi in the mother channel: on the left we have the rate of change of Qi; the first term on the right accounts for the homeostatic effect of the channel on itself, where $Q^0{}_m$ is the normal level of Qi; the second term accounts for the effect (negative in this case) of the child channel on the mother channel (according to the creative cycle law); the third term accounts for the treatment effect (positive in this case) of draining SP-3.

The simplest way to describe the healing process would be as follows:

$$\frac{d\lambda}{dt} = -\left(\frac{dQ_c}{dt} + \frac{dQ_m}{dt}\right),\ 0 < t < T. \tag{5}$$

Equation (5) says that as long as total Qi in the mother and child channels increase, $d\lambda/dt < 0$, the size of the health region is increasing; see equation (1).

Yin Yang balancing channel therapy

We adopt the following simplified model of the process of interaction of the extraordinary vessels with the 12 channels: treatment of either of the Yin pairs of extraordinary vessels, treatment of PC-6 and SP-4 in our case, redistributes the Qi between the six Yin channels so that the distribution is more equal or balanced. Treatment of either of the Yang pairs of extraordinary vessels, treatment of TW-5 and GB-41 in our case, redistributes the Qi between the six Yang channels so that the distribution is more equal or balanced.

We use the notation: Q_1, Q_4, Q_5, Q_8, Q_9, Q_{12}, for Qi in the following Yin channels: lung, spleen, heart, kidney, pericardium, liver. A mathematical model for the treatment, in the case of, say, the spleen channel, is as follows (assume that $t_0 << T$, and that all quantities in the equations below are positive):

$$\frac{dc_{i,4}}{dt} = b_{i,4},\ c_{i,4}(0) = 0,\ i = 9,1,5,12,8,\ 0 < t < t_0,$$

$$\frac{dQ_4}{dt} = c_{9,4}(Q_9 - Q_4) + c_{1,4}(Q_1 - Q_4) + c_{5,4}(Q_5 - Q_4) + c_{12,4}(Q_{12} - Q_4) + c_{8,4}(Q_8 - Q_4),\ 0 < t < T, \tag{6}$$

where $b_{i,4} > 0$, $c_{i,4} \geq 0$ are the coefficients. The first set of equations describes turning on the process of balancing Qi in between spleen and

other five Yin channels during the time interval $[0,t_0]$ of the treatment. The second equation describes the above process of balancing in the case of the spleen channel. Essentially, it says that the amount of Qi, Q_4, in the spleen channel changes so as to equilibrate Qi between the spleen and the other five Yin channels. For example, $Q_9 - Q_4 > 0$ would lead to increase of Q_4, and hence to the decrease of $Q_9 - Q_4$, etc.

The simplest way to describe the healing process, say, for the spleen channel, would be as follows:

$$(7) \qquad \frac{d\lambda}{dt} = \frac{Q_4 - Q_4^0}{|Q_4 - Q_4^0|} \frac{dQ_4}{dt}, \quad 0 < t < T.$$

Equation (7) says that as long as Qi in the spleen channel approaches its normal steady state Q^0_4, $d\lambda/dt < 0$, ie, the size of the health region is increasing; see equation (1).

Linking the Qi-based models with electrodermal measurements

Selecting appropriate measurement technologies at Step 5 to test the models we have developed can be nontrivial, because insufficient data yet exist to clearly validate the selection of any one technology, as each itself still needs to be researched further. Still, in spite of these difficulties, we can suggest a number of measurement technologies that might be useful at Step 5. Electrodermal skin measurements at the acupuncture points and channels have already been used, and many prove useful; magnetic measurements of points and channels could also prove useful; certain biochemical or neurological tests may also prove useful. However, before a technology is used at Step 5, and its measurements are taken as reliable indicators of the models tested from Steps 1 to 3, it will need to be validated, and the theoretical linkages of Step 4 will also need to be examined.

Equations (1)–(7) are written in terms of the quantitative variable Q, Qi. However, no simple relation is presently known between Qi and various material parameters of the body. However, one of us (WAT) has proposed a hypothetical model of how Qi-flow in acupuncture channels creates a magnetic vector potential distribution, $\bar{A}$ along the physical locus of the channel (Tiller, 1989; 1997) and at a magnitude proportional to the Qi magnitude. From one of the two basic electrodynamic equations, this $\bar{A}$ generates an electric field, $\bar{E}$ given by

$$(8) \qquad \bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}$$

oriented along the channel (V = the electromagnetic potential distribution along the channel). This $\bar{E}$ pumps electrolyte ions along the channel toward the surface acupuncture point (A.P.) until the space charge produced by V produces a stationary condition. This leads to a surface ion concentration that increases exponentially with $\bar{E}$. Thus, the A.P. conductivity, σ_{AP}, should be given by an equation of the following form

$$(9) \qquad \sigma_{AP} = \sigma^0_{AP}(1 + e^{eE(z-z0)/kT}) \equiv \sigma^0_{AP} + \beta Q,$$

where β is some complex parameter that may be frequency dependent.

By such reasoning, a connection has been forged between Qi and the electrical impedance, z_{AP}, of skin at an A.P. This allows one to access the available data on skin impedance measurements as a possible vehicle for testing equations (1)–(7), and (9) (Motoyama, 1980; Rosendal 1943; Tiller, 1982, 1989; Voll, 1975). Measurement of A.P surface electrical potential and differential impedance between an A.P. and its nearby non-A.P. skin could also be used to provide additional information on this issue.

Past studies of human z_{AP} in the 0- to 1-MHz range revealed the presence of two electric charge transport processes acting in series (Tiller, 1989). Each of these processes has a unique relaxation time τ_i with $\tau_1 >> \tau_2$. The τ_1 process involves, among other things, carrier conduction along cellular interstices in the stratum corneum ($\tau_1 \approx 10 - 60$ sec). The τ_2 process involves charge transport across epidermis/dermis basal membrane ($\tau_2 \approx 5$–30 ms). It is also possible that more rapid relaxation processes associated with enzymatic activity ($\tau_3 \approx 1$ nanosec) and hydrogen bonding changes ($\tau_2 \approx 1$ terasec) in the tissue fluids of the skin cells will be measured in the future.

Most of the commercial z_{AP} measurement de-

vices in use today utilize the τ_1 process and the results are strongly dependent on (1) the use of constant voltage devices; (2) on the magnitude of the applied voltage; (3) the specific material used as an electrode; (4) the area of the electrode; (5) the humidity and the temperature; and (6) the applied pressure and the electrode contact angle with the skin (Falk et al., manuscript submitted; Margolin et al., 1997; Tiller, 1989). This technique also allows unconscious level processes acting in the operator to influence the measurement via the pressure/angle variation (Tiller, 1989). In many cases, at least one of us (WAT) feels that this leads to superior diagnostic ability on the part of the operator, because an additional information channel is being accessed. Only the Motoyama AMI device uses the τ_2 process and, because of the small time constant, is much less subjective than the τ_1 devices (Tiller, 1989). The B.P. (before polarization) values for A.P.'s, as measured via AMI, are thought to have a connection to Qi.

Many of these device techniques use a small electrode ($\approx$ several mm^2 in area rather than a large electrode ≥ 1 cm^2) and thus measure z_{AP} as distinct from z_{skin} (which may be penetrated by one or more A.P.'s). Experimentally, one finds that $z_{AP} \approx (2 - 20) \times z_{skin}$, depending upon humidity, temperature, hypnogogic state, etc.

In closing this section, we point to a related research direction that may be important to include. This is that the possible correlation between z_{AP} at specific A.P. points and specific electrophysiological measurements could offer (Tiller, 1989) a powerful pathway for understanding traditional concepts (e.g., electrocardiogram measurements via Holter tape recordings versus z_{AP} for points on the heart channel or respiration system electrical recordings versus z_{AP} for specific lung channel points). This type of correlation measurement may not only connect organ functions to specific A.P.'s but could also further the testing of alternative therapies and their relation to traditional concepts.

CONCLUSIONS

We have proposed a five-step process for testing traditional explanatory frameworks of acupuncture that, we believe, could allow a bridge between the diverse traditional concepts and frameworks, and rigorous methods of testing them. We illustrated the diversity of traditionally based systems of acupuncture with a case study, showing three quite different approaches to assessing and treating that patient. Next we developed a general theoretical construct of health and disease that serves as a linkage between the various explanations and approaches, and general physiological functioning. Then, as illustrations of how our mathematical modeling approach can be used, we developed minimal mathematical models of each of the three diagnostic assessments and treatments, showing how each can be related back to the general health model we developed. Finally we briefly discussed measurement technologies, especially electrodermal measurements, focussing on theoretical considerations and methodological issues involved in the use of such measurement techniques.

We emphasize that the main thrust of this article is to introduce a modeling approach that, as the authors believe, provides a precise language and framework for initializing basic research in TBSAs: beginning to formulate testable hypotheses and design experiments to test them. Our approach is a Western scientific approach for dealing with very diverse and poorly understood phenomena, for which no coherent framework has yet been established: Qi, subtle energies, traditional acupuncture concepts, and their hypothesized holistic properties.

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